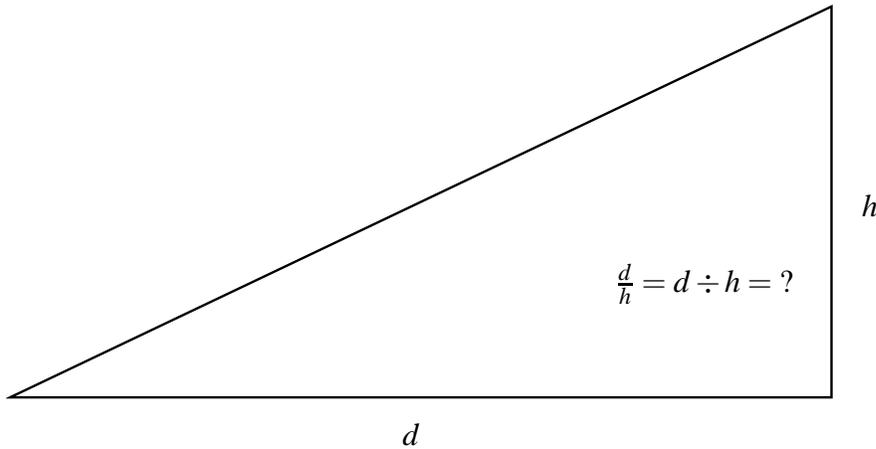
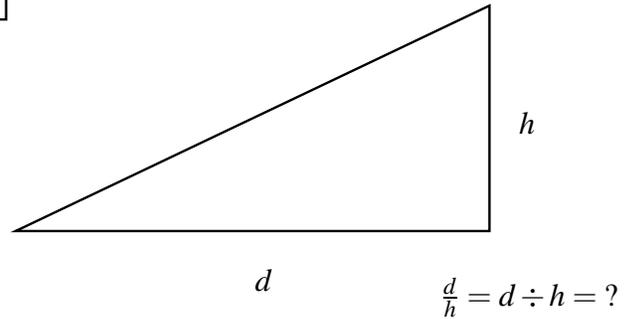
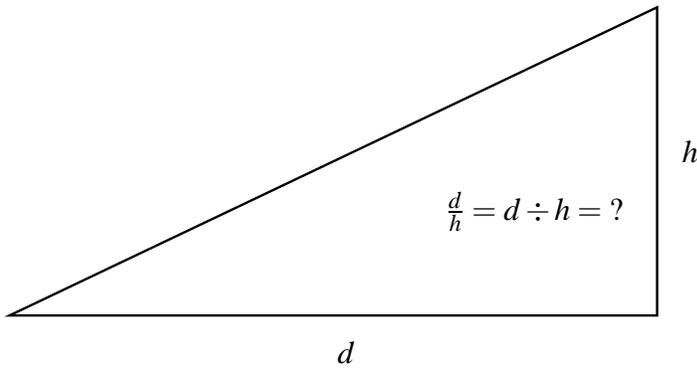


# Clinometers for Measuring Tall Objects \*

October 22, 2005

## Opposite:Adjacent

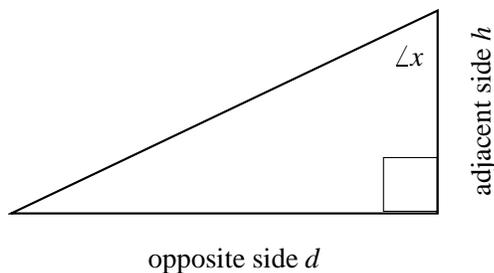
In the 3 triangles below, measure the distance  $h$  and  $d$  as accurately as you can (you should probably use millimeters). For each triangle calculate the ratio of the length  $d$  to  $h$  or  $\frac{d}{h}$ .



Do you notice anything special about the quotient? It should have been very close to the same number (2 or 2.1). This is because the three triangles above are all *similar*, they are simply shrunken or magnified copies of each other.

\*derived from <http://www.cimt.plymouth.ac.uk/projects/mepres/book9/y9s15act1.pdf>

# Tangent

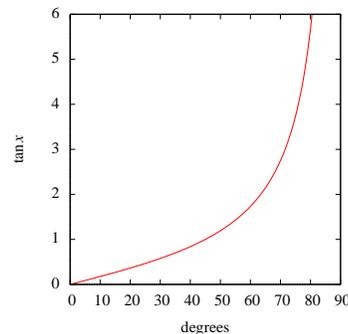


This quotient has a very special name, it is called the *tangent of  $x$* , usually written as ' $\tan x$ '. What is the  $x$ ? It is one of the angles in the triangle – it is the angle *opposite*, or not touching, the leg  $d$ . Leg  $d$  is called the *opposite side* of  $x$ .

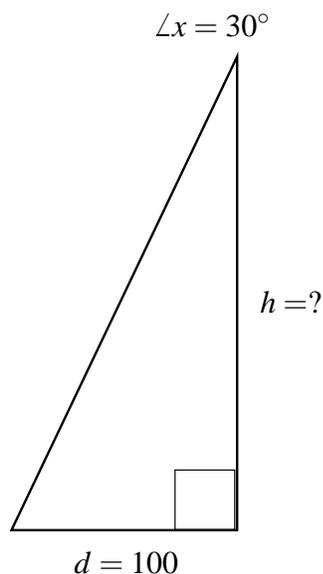
Tangents only exist within *right triangles*, and the value of the tangent within a right triangle depends on the angle  $x$ .

Suppose we took an entire day and walked around finding all the right triangles in our school. Everything from the doorways to the square garbage cans to the pieces of paper we write on. Whenever we found a new right triangle, we would measure three things: the angle  $x$ , its opposite leg  $d$  and its adjacent leg  $h$ . We could create a table with two columns, the value of  $x$  and the ratio  $d : h$ . If we plotted these points, they wouldn't fall on a straight line, the relationship between angle and tangent is *curvilinear*.

$x$	$\tan x = d \div h$
$10^\circ$	0.18
$30^\circ$	0.58
$45^\circ$	1.00
$60^\circ$	1.73
$80^\circ$	5.67



## Using a $\tan x$ Table



Could we use this new table of  $\tan x$  values? Of course we could! Suppose our homework assignment is to find the length  $h$  of a triangle when we only know the opposite side and its angle. We can use the tangent  $30^\circ$  value we already know from our table and setup an equation. We substitute the known values for  $\tan x$  and  $d$  in Eq. 1, multiply both sides by  $h$  in Eq. 2, and finally solve for  $h$  with division by 0.58 in Eq. 3.

$$\begin{aligned} \tan x &= d \div h \\ &= \frac{d}{h} \\ 0.58 &= \frac{100}{h} \end{aligned} \tag{1}$$

$$0.58h = 100 \tag{2}$$

$$h = 172.4 \tag{3}$$

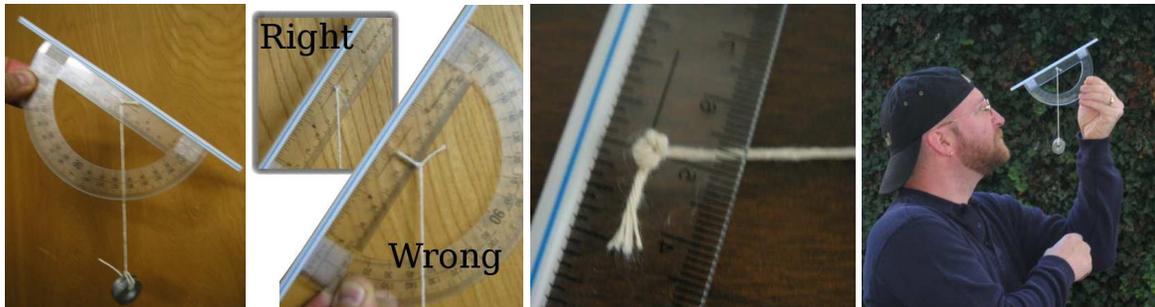
## Calculator Tangents

You can find the tangent of any angle  $x$  on your calculator by using its tangent function. The key probably looks like this:  $\boxed{\text{TAN}}$  or  $\boxed{\text{tan}}$ . On many standard scientific calculators, you simply enter the angle then press the tangent key and the value appears in the display window. If you get the answer  $\boxed{-6.405331197}$ , then your calculator is in *radians* mode. Consult your calculator's documentation for how to use degrees mode instead.

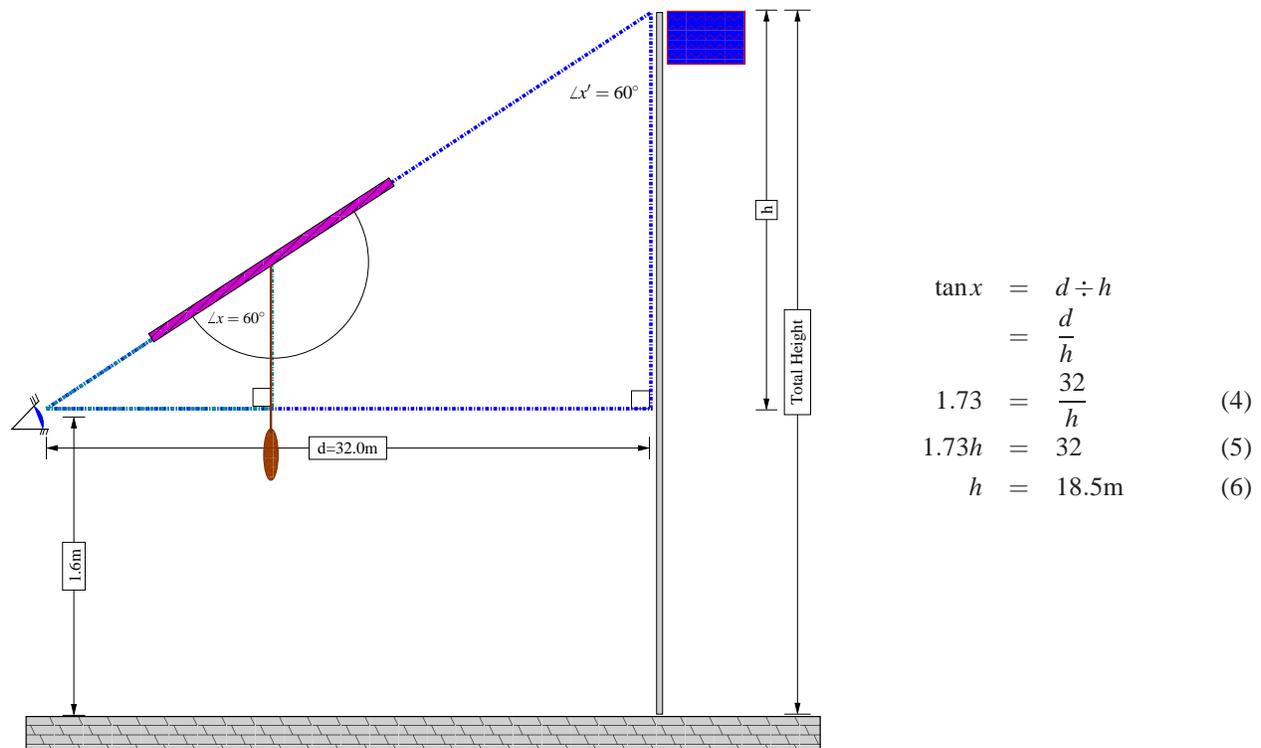
Keys	Display
$\boxed{3} \boxed{0}$	$\boxed{30}$
$\boxed{\text{tan}}$	$\boxed{0.577350269}$

# Clinometer

We can use the tangent of right triangles to easily measure the height of tall objects. To do this we will build a specialized device called a *clinometer* using a protractor, straw, string, and a weight. Tie the weight to end of a 20cm length of string. Attach the other end of the string to the hole at the center of the protractor's circular scale. There are a number of ways to attach the string onto the protractor, *be sure that the string falls freely from the protractor's center, if it doesn't inaccurate measurements will result.* Tape the straw along the flat side of the protractor. Make sure the straw is parallel to the flat edge of the protractor for the full length of the protractor.



It is easiest to use the clinometer with a partner. One person will sight the top of the measured object through the straw, the other will read and record the angle measurement on the protractor. When you measure an object, two similar right triangles are created. When you measure the  $\angle x$  on the protractor, you are also measuring the  $\angle x'$  at the top of the measured object! If you know the distance  $d$  to the object being measured, then you can calculate the object's height  $h$ .



If  $\angle x = \angle x' = 60^\circ$  and the distance  $d$  is 32m, then we can calculate  $h$ . We substitute the known values for  $\tan x$  and  $d$  in Eq. 4, multiply both sides by  $h$  in Eq. 5, and finally solve for  $h$  with division by 1.73 in Eq. 6.

However, this is only the height of the flagpole *above* the eyes of the person who sighted through the clinometer. For the most accurate measurement of an object's height, we must add the distance between the ground and the measurer's eyes. My eyes are 1.60m from the ground, so the final calculated height of the flagpole is

$$\text{Total Height} = 18.5 + 1.60 = 20.1\text{m}$$

## Follow Up Questions

1. What types of tall objects could a clinometer be used to measure?
2. What types of objects would be difficult to measure with a clinometer?
3. Can a clinometer be used to measure the *depth* of something? Or is a clinometer only good for measuring the height?
4. Suppose you're measuring tall objects with a protractor and string clinometer on a windy day. So windy that it is difficult to get an accurate measurement of  $\angle x$ ; the wind is blowing the string around by  $\pm 3^\circ$ .
  - Will your measurements of very tall objects be affected less, equally, or more than your measurements of short objects?
  - Will your measurements of far away objects be affected less, equally, or more than your measurements of near objects?
  - Can you summarize the conclusions of the previous two questions into one rule of thumb for using a clinometer?
5. How can the homemade clinometer design be changed so that measurements made with it are less susceptible to error on windy days?