calculating antenna bearings

for geostationary satellites

Using a pocket calculator to accurately determine antenna bearings and range to geosynchronous satellites

Many members of the amateur community have recently "discovered" the geosynchronous communications satellite. This surge of interest is in response to a program of weather satellites now affording an outstanding view of the earth from a 22,000 mile (35,000 km) vantage point.[1] It is further spurred by the recent availability of low cost, high quality microwave receiving equipment which enables the individual to recover not only this weather data,[2] but promises reception of direct-broadcast closed-circuit TV programs.

Although a great deal of material has been published on generating tracking information for polar orbiting spacecraft such as Oscar 7,[3,4] as well as the highly elliptical orbit planned for AMSAT Phase III,[5] little has appeared in the amateur magazines regarding the geostationary, or earth-synchronous, orbit. Peter Thompson has published a set of generalized equations which could be applied to tracking satellites in a variety of orbits,[6] but the calculations are unnecessarily cumbersome when considering the simple geostationary case. Ralph Taggart has outlined an appealing method for estimating azimuth and elevation bearings with a globe and string,[7] but like most graphical plotting techniques, this one offers resolution limited by the finite size of the globe. The equations presented here can be readily solved on a pocket calculator and afford bearing accuracy which is limited only by your ability to point the antenna.

*If the computed value for L is greater than 180°, correct it by subtracting 360°; if the computed value is less than −180°, correct it by adding 360°.

If an imaginary line is drawn which connects the center of a satellite with the center of the earth, that line intersects the earth's surface at a location known as the Sub-Satellite Point or SSP (**fig. 1**). Generating *azimuth* angles to a satellite is a function only of the location of the observer and the sub-satellite point, and is completely independent of the satellite's altitude (altitude does, of course, determine the elevation bearing, which is computed later). Hence, the azimuth data is based solely upon terrestrial coordinates; the law of cosines for determining distance and bearing coordinates in a great circle will apply.

Jerry Hall reported the relationships in his article on terrestrial great-circle computations;[8] those same equations may be used to calculate satellite azimuth angles:

$$\cos D = \sin A \sin B + \cos A \cos B \cos L \qquad (1)$$

$$\cos C = \frac{\sin B - \sin A \cos D}{\cos A \sin D} \qquad (2)$$

Where $A =$ latitude of Point 1 (the observer)

$B =$ latitude of Point 2 (the sub-satellite point)

$L =$ longitude of Point 1 *minus* the longitude of Point 2*

$D =$ distance between Point 1 and 2, degrees of arc

$C =$ true bearing if L is positive

or $360 - C =$ true bearing if L is negative

Interestingly enough, the geostationary satellite presents a special case because the latitude of the sub-satellite point is always approximately zero degrees; under these conditions B in the above formulas is 0.

Since $\sin B = 0$ and $\cos B = 1$, these equations simplify to:

$$\cos D = \cos A \cos L \qquad (3)$$

$$\cos C = -\frac{\sin A \cos D}{\cos A \sin D} \qquad (4)$$

By trigonometric identities, **eq. 4** further simplifies to

$$\cos C = -\frac{\tan A}{\tan D} \qquad (5)$$

Thus, by solving **eqs. 3** and **5** for D and C, respectively, antenna azimuth information can be found.

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calculating elevation

If a straight line is drawn between a satellite and the center of the earth, and another straight line is used to connect the observer with the earth's center, the angle formed at the intersection of these two lines represents D, the distance between the observer and the sub-point, in degrees of arc, just computed (**fig. 2**). If the observer happened to be situated very near the center of the earth, angle D would relate directly to elevation angle; that is, D would be the angular *displacement from vertical* for aiming the antenna. Since elevation angles are generally specified with respect to the horizontal, *uncorrected* elevation data is computed from

$$EL\ (assumed) = 90 - D \tag{6}$$

This assumes that the observer is located at or near the center of the earth. Since this is obviously impossible, let's consider corrections to the assumed elevation which would apply to an observer on the earth's surface.

If **eq. 6** is used, the error is negligible when R_s, the radius of the satellite's orbit, is at least two orders of magnitude greater than r_e, the radius of the observer's orbit (the radius of the earth). Thus, **eq. 6** is used in determining elevation information for radio astronomy, where the object being tracked is very far from earth. Under these conditions the observer *is* near the earth's center, *relative* to the object being tracked. **Eq. 6** has also been used by EME operators for tracking the moon; although the radius of the lunar orbit is only about 50 times the radius of the earth, elevation data calculated from **eq. 6** is correct to within a fraction of a degree.

It is interesting to note that in both of the above applications, the location of the object being tracked is generally specified in Greenwich Hour Angle (GHA) and declination. These coordinates correspond exactly to the longitude and latitude, respectively, of the sub-satellite point shown in **fig. 1**.

What do you do to correct **eq. 6** for geostationary satellites, whose orbital radius is *not* significantly

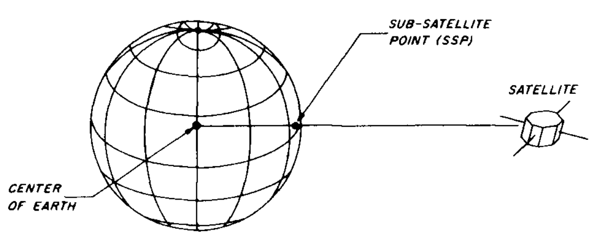

fig. 1. When a straight line is drawn from the center of the earth to a satellite, the intersection of the line with the earth's surface is known as the Sub-Satellite Point or SSP. The SSP is used to calculate the azimuth bearing to the satellite.

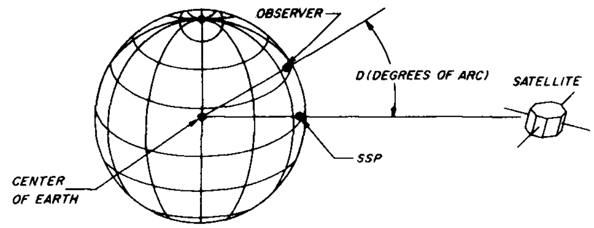

fig. 2. The observer's position on the earth's surface, in relation to the imaginary straight line shown in fig. 1, yields the angle D, which is used to calculate antenna elevation angle as discussed in the text.

greater than the radius of the earth? A correction formula published by L. R. Larson[9] applies:

$$\tan EL = \tan (90 - D) - \frac{1}{K \cos (90 - D)} \tag{7}$$

where K is the ratio of satellite orbital radius to earth radius

$$K = R_s / r_e$$

Since the mean radius of the earth is approximately 3444 nautical miles, and the mean radius of geostationary orbits is approximately 22,766 nautical miles, I use the value $K = 6.61$ when working with geostationary orbits. In the interest of minimizing computational steps, **eq. 7** may be restated

$$\tan EL = \frac{\sin (90 - D) - (1/K)}{\cos (90 - D)} \tag{8}$$

where $$\frac{1}{K} = r_e / R_s = 0.1513 \tag{9}$$

Eq. 8 may be further simplified

$$\tan EL = \frac{\cos D - (1/K)}{\sqrt{1 - (\cos D)^2}} \tag{10}$$

calculating slant range

System performance predictions, which include link calculations of signal margin, require a knowlege of the exact distance from the ground station to the orbiting satellite. One convenient equation for determining slant range is stated as follows[10]

$$Range = R_s \times \sqrt{1 - 2(1/K) \cos D + (1/K)^2} \tag{11}$$

If you want to calculate the range in nautical miles, use 22,766 nautical miles for R_s. If you would rather compute the range in kilometers, use 42,166 km for R_s. The ratio K, of course, is the same regardless of the units used to express R_s and r_e.

Example. My station is located approximately 37.3° N, 121.9° W;* I wish to receive signals from SMS-2,

*If you know your latitude and longitude to within 1/10 degree, you have pinpointed your location to within about 6 miles!

in geostationary orbit above 135° W. The longitude difference is thus $121.9 - 135 = -13.1°$. Note that L is *negative*; this information will be used later.

The distance D is found from **eq. 3** to be 39.22°

From **eq. 5**, C is found to equal 158.99°. Since L is negative, true azimuth bearing is $360 - C$, or 201.01°.

From **eq. 10**, the corrected elevation bearing equals 44.61°.

Eq. 11 yields a slant range of 20,215 nautical miles.

These computations agree well with my actual experience in tracking SMS-2.

calculator programs

The above relationships, with their conditional branching requirement depending on the sign of L, are ideal candidates for solution on a programmable calculator. I use a Hewlett-Packard 25 calculator which, with its 49 available program steps, demands a degree of programming ingenuity. After several false starts, I came up with the program listed in **table 1**, which appears to be valid for any point on the Earth*. Note that southern latitudes and eastern longitudes must be entered as *negative* numbers.

The following steps show how to use the program.

1. Key in the program, switch to RUN, depress f PGM.
2. Store the constant 180 in Register 3 (180 STO3)
3. Key in 3444 (for km, key in 6368)
4. ENTER
5. Store 22766 in Register 4 (for km store 42166)
6. Depress divide (÷) key
7. Store answer in Register 2
8. Store latitude of observer in Register 0
9. Depress f COS keys
10. Store answer in Register 1
11. Key in longitude of observer.
12. ENTER
13. Key in longitude of SSP.
14. Depress R/S key
15. Calculator displays range in nautical miles (or kilometers)
16. Depress rolldown key (R↓)
17. Calculator displays elevation angle in degrees
18. Depress RCL 6

*In reviewing this manuscript WB8DQT observed that the program will display ERROR if the observer is located directly under the satellite (on the equator at the SSP). Under these conditions the required azimuth bearing is undefined (the bearing is 90 degrees and the slant range is equal to $R_s - r_e = 19,322$ *nautical miles* [35,798 *km*]).

HP-25 Program

SWITCH TO PRGM MODE, PRESS [f] [PRGM]. THEN KEY IN THE PROGRAM.

DISPLAY LINE	DISPLAY CODE	KEY ENTRY	X	Y	Z	T	REMARKS
00							
01	41	−					
02	23 05	STO 5					
03	14 06	TAN					
04	32	CHS					
05	24 00	RCL 0					
06	14 04	SIN					
07	71	÷					
08	15 06	TAN⁻¹					
09	24 03	RCL 3					
10	51	+					
11	23 06	STO 6					
12	24 00	RCL 0					
13	15 51	x ≥ 0					
14	13 17	GTO 17					
15	24 03	RCL 3					
16	23 51 06	STO+6					
17	24 06	RCL 6					
18	24 03	RCL 3					
19	02	2					
20	61	x					
21	14 51	x ≥ y					
22	13 24	GTO 24					
23	23 41 06	STO−6					
24	24 05	RCL 5					
25	14 05	COS					
26	23 61 01	STO x1					
27	24 01	RCL 1					
28	24 02	RCL 2					
29	41	−					
30	01	1					
31	24 01	RCL 1					
32	15 02	x^2					
33	41	−					
34	14 02	$\sqrt{x}$					
35	71	÷					
36	15 06	TAN⁻¹					
37	01	1					
38	24 02	RCL 2					
39	24 01	RCL 1					
40	61	x					
41	02	2					
42	61	x					
43	41	−					
44	24 02	RCL 2					
45	15 02	x^2					
46	51	+					
47	14 02	$\sqrt{x}$					
48	24 04	RCL 4					
49	61	x					

MEMORY REGISTERS
R0 Latitude of observer
R1 D
R2 0.1513 (1/K) (re/Rs)
R3 180
R4 Rs (22766 naut miles)
R5 Longitude of observer minus Long. of sat.
R6 Azimuth
R7

table 1. HP-25 program for calculating antenna azimuth and elevation angles and slant range to a geostationary satellite from any point on earth. Note that southern latitudes and eastern longitudes must be entered as negative numbers.

19. Calculator displays azimuth bearing in degrees
20. To perform further calculations, return to **Step 8**.

I would like to thank Glenn Thomas, WB6YZI, for his assistance in evaluating these computations. Glenn works at a satellite tracking facility and says, "Look-angles for geosynchronous satellites are my stock in trade." Believe me, he really makes it all look ridiculously easy!

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